

# Case-based reasoning foundations

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## Abstract

A basic observation is that Case-based Reasoning has roots in different disciplines: Cognitive science, knowledge representation and processing, machine learning and mathematics. As a consequence, there are foundational aspects from each of these areas. We briefly discuss them and comment on the relations between these types of foundations.

## 1 Introduction

The foundations of an area are concerned with the basic elements underlying the problems, methods, results and applications of the field. Sometimes this is easy to determine, e.g., when one is concerned with the foundations of logic. In case-based reasoning (CBR) it is not as simple because CBR is in the intersection and the interest of different disciplines of a rather heterogeneous nature. Each discipline has its own roots, its own aspects and its own foundations. That makes an overview on the foundations of CBR a complex task, and one that we will not attempt to address in detail. Instead, we will mention a set of basic problems tackled, related to essential foundational areas. A general overview is given in (Kolodner & Leake 1996).

We will discuss foundational issues of CBR related to the influence from cognitive science, from basic AI issues of representation and reasoning, from machine learning, and from mathematics. The areas clearly overlap, but they still capture significant foundational perspectives on case-based reasoning.

## 2 Cognitive science

CBR concerns the capturing and utilization of specific experiences. The major influence of cognitive science on CBR is centered around the terms *experience*, *memory* and *analogy*. These concepts have their roots outside of computer science. Tulving's (1972) distinction between episodic and semantic memories in human reasoning is central, as is Smith and Medin's (1981) characterization of the exemplar view in concept definition. In the exemplar view, a concept is defined as the contents of its set of exemplars. Schema-oriented memory models have a long tradition in psychology (Bartlett, 1932). Many features of Bartlett's schema theory occur in Schank's theory of dynamic memory (Schank, 1982). The dynamic memory theory has memory organization packets (MOPs) as elements that have had quite an impact on CBR. They were further refined into E-MOPs (episodic MOPs) by Kolodner (1983) in her influential work on dynamic memory in the Cyrus system. This work led to a range of theories and systems on the utilization of specific experiences in understanding, problem solving, and learning (Kolodner & Riesbeck, 1986; Leake, 1988).

CBR research paid attention to analogical reasoning in particular in the early phases of the field. The cognitive foundations of analogy had a long history. In some sense one can state that CBR is analogical reasoning, but that ignores the fact that analogy typically reasons across domains while CBR reasons inside one domain. Derivational analogy was discussed in (Carbonell, 1986): A basic theory was concerned with analogical mappings (Gentner, 1983); for a discussion see (Keane, 1994). These mappings are related to adaptation rules but several differences exist, e.g., humans usually do not often use adaptation. The study of analogical reasoning in humans has been a focal issue in cognitive science, but less work has focused on computational or mathematical aspects of analogy. Some computational models of human analogical reasoning, though, such as computational models of relational mapping (Falkenhainer *et al.*, 1990), have had significant impact on CBR. Other important contributions have been made by Ken Forbus, e.g. (Forbus, 2001).

### 3 Knowledge representation and reasoning

As one of the main AI paradigms, CBR represents knowledge and reasons with it. In CBR a body of cases represent first class knowledge, upon which reasoning methods for similarity assessment, case adaptation, and learning of new cases are applied. The variety of ways in which CBR systems were developed in the first ten years of the field is described in Kolodner's (1993) CBR text book. Despite the many different appearances of CBR systems the essentials of CBR is captured in a surprisingly simple and uniform process model, the CBR cycle (Aamodt & Plaza, 1994). However, this is only a basic model, and for more specific considerations it has to be refined. Several refinements have been proposed; they either add elements to the cycle or split it into subcycles, for example adding a maintenance step (Roth-Berghofer, 2002).

There has been no attempt to create new logic formalisms and calculi for CBR; it sufficed to use and adapt existing formalisms from within AI, such as description logics (e.g., Díaz-Agudo & González-Calero, 2001), and feature terms (Plaza, 1995). Established KR methods are generally suitable as a basis for representing cases as well as general knowledge used for support of the case-based processes (e.g., Koton 1988; Aamodt, 1994). Particular requirements for CBR are usually dealt with as they arise; more generic issues of knowledge representation are seldom addressed. An example is the rule knowledge that is often used in adaptation. Although rules have been thoroughly investigated in the literature on rule-based knowledge systems, little generic work has been done with respect to rule systems for adaptation.

The concept of knowledge containers (Richter 1995; 2005) has thrown a new light on knowledge modeling in CBR. The knowledge containers are the vocabulary, the similarity measure, the case base, and the adaptation rules. The case base plays a special role because the cases can be entered without understanding them. The main point is that knowledge can be shifted between containers (their content is not invariant), which can be modeled using a learning process. In addition, the shifting can be done manually without the support of a learning method.

An important representational issue is how to index a set of cases in a meaningful and efficient memory structure. This has both a dimension of cognitive plausibility, and one of computational efficiency. While cognitive issues were focused in the work mentioned on dynamic memory, computational issues have been focused by leaning on methods from basic computational science, i.e., algorithms and data structures. The main data structures that occur in CBR are the traditional data structures also used in data base technology. In addition to well-known structures such as discrimination trees and decision trees, an important tree structure is k-d trees, for which complexity properties and related questions for CBR have been investigated to some extent (Wess *et al.*, 1994). Surely, not all problems concerning k-d trees have been solved. Other types of indexing trees have also been discussed but there are even more open problems such as which trees are superior in which context.

#### 4 Machine learning

CBR has close relations to machine learning in several respects although it can hardly be regarded as a subfield of machine learning. The CBR cycle embodies an approach to machine learning, characterized by “lazy” learning, i.e., simply store a specific instance at learning time, postponing the main inductive step until problem solving. Traditionally, machine learning has focused on concept learning, i.e., inducing a general concept definition based on a set of positive and negative examples. CBR methods that address the learning of concept definitions for classification tasks are sometimes referred to as *exemplar-based*, or *instance-based*. Kibler and Aha (1987) showed that concept learning could be achieved by storing the exemplars presented to the learner, and subsequently, when an unknown example is presented by assigning to it the concept class of the best matched exemplar. Others showed that case matching could be improved by adding a model of general domain knowledge for use in the matching process (Porter *et al.*, 1990). The knowledge-poor approach, with no general knowledge and a global similarity metric, was developed into the IBL series of algorithms for instance-based learning (Aha *et al.*, 1991), to which a lot of later work in case-based learning has been related.

#### 5 Mathematical foundations

Foundational mathematical investigations are concerned with principal properties of a CBR system, e.g., what is in principle possible or impossible to achieve and under which conditions? The influence of mathematical methods is centered around similarity measures and learning. As a consequence, most approaches deal with questions of approximation theory. Measures are mathematical functions that describe intended objects, and mathematical functions for approximating utility functions. This is a principal problem of approximation theory: How good can a subset of functions approximate functions of a larger class? Corresponding questions have been treated extensively in the area of neural networks (see (Hammer, 2002)). The main class of similarity measures is defined as linear functions with coefficients that can be chosen. This class has been widely investigated. Emphasis was put on learning and determining the coefficients (the weights) so that one now has a good overview of the major facts.

A question that remains open is which classes of measures can be approximated by weighted measures or, in other words, in which function spaces is this class dense. Here investigations have still to be done. The corresponding density problem arises for case bases. For the previously mentioned IBL algorithms, for example, we have a convergence theorem stating that the IB1 algorithm generates a case base that is dense in the sense that except for a set of arbitrarily small measures each point is arbitrarily close to a generated point with an arbitrarily high probability. Many extensions exist, e.g., with respect to noisy data or cost sensitive classification. In this connection one has to mention that there are many (in fact, too many to name them here) mathematical results on probability in CBR in various directions.

Another problem is concerned with minimality questions of a CBR system. For case-based classifiers this has been investigated in (Globig *et al.*, 1997). Typical results with respect to the learnability of classifiers concern “how many examples do we need?” or “how many values are necessary for a similarity measure?”. Furthermore, one would like to know under which conditions are polynomial local measures sufficient for an approximation in the limit of a given classifier.

#### 6 Conclusion

CBR is founded on a set of well-established scientific fields. An important point for further research is to study in a deeper fashion the relations across the foundational areas described above – and possibly others. CBR seems to be one of the few areas of computer science where so many foundational aspects meet, and this represents a particular challenge for establishing a foundational platform in the field. Such investigations could have a positive impact on CBR

from an integration point of view. A major point is in how far the foundational insights that come from the different areas conflict with each other. For instance, how far do requirements coming from cognitive science restrict the applicability of computational methods? Another example could be to get more insight into particular foundational questions, such as the principle relation between rules and measures.

In summary, the foundational picture of case-based reasoning is quite promising, but there is still much to do. A deeper insight is not only of academic interest but also important for application issues. If CBR is to become an established engineering discipline, then one should examine other engineering sciences, such as electrical or mechanical engineering, as well as more complex disciplines such as environmental engineering. Most engineering disciplines rely on solid foundations of mathematics, physics, or system theory that allow experimentation, innovation, and the potential for new applications. In order to become an established engineering discipline it would help CBR to examine other engineering disciplines. For getting innovative applications this seems to be very useful if not necessary. We are still some way from a unified foundation of CBR. A good continuation, which should reveal new insight, would be an in-depth study of the relationships and dependencies among the foundational areas.

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